REPORT #1: ESTIMATION OF MAXIMAL CRITICAL HEALTH FACILITIES DEMAND FOR COVID-19 OUTBREAK IN SANTIAGO, CHILE

Authors: Alonso Cancino, Pedro Gajardo, Rodrigo Lecaros, Claudio Muñoz, Héctor Ramírez, Jaime Ortega

Institutions: Centro de Modelamiento Matemático - CMM (Universidad de Chile), Group of Analysis & Mathematical Modeling Valparaiso - AM2V (Universidad Técnica Federico Santa María)

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Abstract. In this document we propose a compartmental epidemiological model in order to estimate the maximal demand of critical health care facilities, needed by a city (Santiago, Chile) during a COVID-19 outbreak. Considering as control variables the rates of contacts with people in different stages of the disease, we report the variation of maximal demands when different mitigation strategies are applied. We consider two classes of these strategies: (i) testing, isolation and contact tracing; (ii) distancing measures. In this report we only give results for simulations of strategy (i).

1. Model formulation

The disease spread within a particular contaminated city is modeled by using a deterministic compartmental model (see, for instance [1]). We use this approach in this report because the simplicity and the rapidity to obtain some results that can be used later for more complex models. We adapt the model proposed in [3] considering as main assumption an isolated city. Thus, the model proposed consists of a compartmental model, where the population is distributed into 7 groups corresponding to different stages of the disease: Susceptible (S), exposed (E), infected (I), recovered (R), hospitalized (H), hospitalized requiring critical services (H^c), and dead (D) as direct consequence of COVID-19. The total population is then

$$N = S + E + I + R + H + H^c. \tag{1}$$

As usual, these groups of stages are called state variables, so the vector of state variables is $x = (S, E, I, R, H, H^c, D)$. We omit to consider N as a state variable because of equality (1).

The control variables to be considered are the actions on the rate of contagious denoted by $u = (u_E, u_I, u_H, u_{H^c}) \in \mathbb{R}^4_+$, where

$$\beta_X = p_X u_X \tag{2}$$

with p_X the probability to be exposed (i.e., infected but asymptomatic) after a contact with a person in the stage X, and u_X is the rate of contact with a person in the stage $X \in \{E, I, H, H^c\}$.

One should have

$$u_E \geq u_I \geq u_H \geq u_{H^c} \approx 0.$$

For this reason, we do not consider u_H and u_{H^c} as control variables, because we will assume these values constant and near to zero.

Now, for each control u_X, with $X \in \{E, I\}$ we consider reference values (to be calibrated) $u_X^{\text{ref}} > 0$. Hence, $u_X(t) \in [0, u_X^{\text{ref}}]$ for all $t \geq 0$, with $X \in \{E, I\}$.

The objective of this study, is to report, for different strategies represented by $u_E(\cdot)$ and $u_I(\cdot)$, the maximal demand of critical health facilities.

The evolution of state variables is described by the following system of ordinary differential equations:

$$(3) \qquad \left\{ \begin{array}{lcl} \dot{S} & = & \mu_b N - S \overbrace{\left(\frac{\beta_E E + \beta_I I + \beta_H H + \beta_{H^c} H^c}{N} \right)}^{\Lambda(x,u):\ \text{rate of contagious}} - \mu_m S \\ \dot{E} & = & S \left(\frac{\beta_E E + \beta_I I + \beta_H H + \beta_{H^c} H^c}{N} \right) - (\gamma_E + \mu_m) E \\ \dot{I} & = & \gamma_E E - (\gamma_I + \mu_m) I \\ \dot{H} & = & (1 - \phi_{IR}) \gamma_I I + (1 - \phi_D) \gamma_{H^c} H^c - (\gamma_H + \mu_m) H \\ \dot{H^c} & = & (1 - \phi_{HR}) \gamma_H H - (\gamma_{H^c} + \mu_m) H^c \\ \dot{R} & = & \phi_{IR} \gamma_I I + \phi_{HR} \gamma_H H - \mu_m R \\ \dot{D} & = & \phi_D \gamma_{H^c} H^c. \end{array} \right.$$

From (1) and (3) we obtain that the dynamics of the total population is

$$\dot{N} = (\mu_b - \mu_m) N - \phi_D \gamma_H H^c.$$

The above system (3) is represented also in Figure 1 below.

2. Parameters

The parameters to be identified (literature and/or calibration) are

$$(4) \qquad P = (p, \mu_b, \mu_m, \gamma, \phi, u^{\text{ref}}) \in [0,1]^4 \times \mathbb{R}_+ \times \mathbb{R}_+ \times [0,1]^4 \times [0,1]^3 \times \mathbb{R}_+^4 \subset \mathbb{R}^{17}.$$

The descriptions of these parameters are the following:

- $p = (p_E, p_I, p_H, p_{H^c})$ are the probabilities of contagious (see (2)) when a susceptible person is in contact with a people in stages E, I, H, and H^c.
- μ_b is the natality rate in the city and μ_m is the mortality rate, both measured in $[\text{day}]^{-1}$;

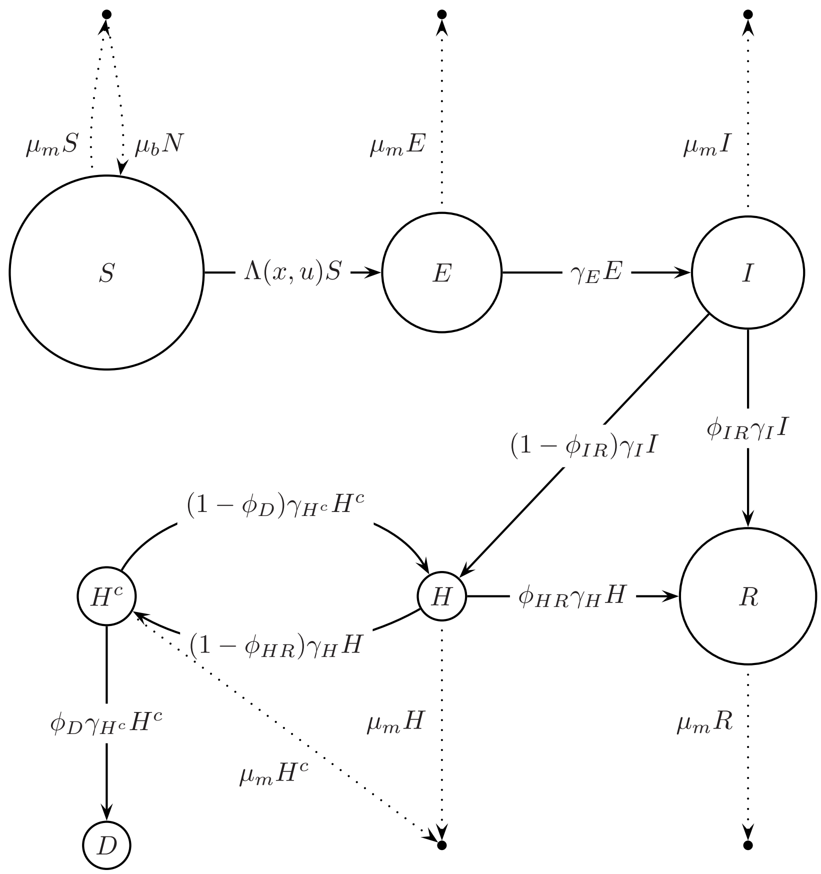

FIGURE 1. **Structure of the mathematical model for the dynamics of COVID-19 in an isolated city.** Each circle represents a compartment. Susceptible individuals (S), and different disease states: exposed (E), infected (I), recovered (R), hospitalized (H), hospitalized requiring critical services (H^c), and dead (D).

- Parameters γ_X measured in $[\text{day}]^{-1}$ are the rate of transition from a disease stage $X \in \{E, I, H, H^c\}$ to the following stage, where γ_X^{-1} represents the mean duration of stage X;
- ϕ_{IR} is the fraction of infected people that recover;
- ϕ_{HR} is the fraction of hospitalized (in normal services) people that recover;
- ϕ_D is the fraction of hospitalized (in critical services) people that die;
- The vector $u^{\text{ref}} = (u_E^{\text{ref}}, u_I^{\text{ref}}, , u_H^{\text{ref}}, , u_{H^c}^{\text{ref}})$ contains references values of rates of contact.

The baseline values and ranges considered for each parameter are indicated in Table 1. The baseline values were obtained after calibration for obtaining a *basic reproduction number* $\mathcal{R}_0 = 2.15$ [2] but associated with the proposed model (3). Ranges of values are taken from literature and consideration of the authors of this report.

For initial conditions we consider the total population of 5.624 millions people, and an estimation of cases until today. The values used are indicated in Table 2.

Notation	Unit	Baseline	Range of values	References
p_E	dimensionless	0.1399	$[0, 0.2]$	modeling team
p_I	dimensionless	0.855	$[0.75, 0.9]$	modeling team
p_H	dimensionless	0.855	$[0.75, 0.9]$	modeling team
p_{H^c}	dimensionless	0.855	$[0.75, 0.9]$	modeling team
μ_b	$[\text{day}]^{-1}$	$3.57 \cdot 10^{-5}$	$[3.4 \cdot 10^{-5}, 3.6 \cdot 10^{-5}]$	INE CENSO 2017
μ_m	$[\text{day}]^{-1}$	$1.57 \cdot 10^{-5}$	$[1.4 \cdot 10^{-5}, 1.6 \cdot 10^{-5}]$	INE CENSO 2017
γ_E	$[\text{day}]^{-1}$	0.185	$[1/16, 1/5]$	modeling team
γ_I	$[\text{day}]^{-1}$	0.179	$[1/16, 1/5]$	modeling team
γ_H	$[\text{day}]^{-1}$	0.116	$[1/16, 1/8]$	[2] / modeling team
γ_{H^c}	$[\text{day}]^{-1}$	0.121	$[1/10, 1/8]$	[2] / modeling team
ϕ_{IR}	dimensionless	0.9515	$[0.95, 0.96]$	[2] / modeling team
ϕ_{HR}	dimensionless	0.665	$[0.65, 0.75]$	[2] / modeling team
ϕ_D	dimensionless	0.385	$[0.3, 0.4]$	modeling team
u_E^{ref}	dimensionless	0.82	$[0, 0.82]$	modeling team
u_I^{ref}	dimensionless	0.32	$[0, 0.32]$	modeling team
u_H^{ref}	dimensionless	0.01	fixed value	modeling team
$u_{H^c}^{\text{ref}}$	dimensionless	0.01	fixed value	modeling team

TABLE 1. Parameters used in model (3), baseline and range values.

State	Value (individuals)
S_0	$5.624 \cdot 10^6$
E_0	1000
I_0	440
H_0	200
H_0^c	4
R_0	100
D_0	1

TABLE 2. Initial conditions for (3), considering the total population of Santiago and an estimation of cases until today.

3. Representation of strategies

One of the objetives is to evaluate the impact of different mitigation strategies in the maximal demand of critical health care facilities. In a first approach, the strategies considered are:

- **Testing, isolation and contact tracing**, represented by decreasing u_E and u_I constantly. That is, $u_X(t) \equiv \alpha_X u_X^{\text{ref}}$ with $\alpha_X \in \{0.1, 0.5, 0.75\}$ for $X \in \{E, I\}$. If the effort to massive tests is not high, then $\alpha_E > \alpha_I$.

- **Distancing measures** (closing schools, universities, quarantines) represented by decreasing u_I and u_E temporally. That is, consider a period of time $[0, T_X]$ where u_X decreases, i.e.,

$$u_X(t) = \begin{cases} \alpha_X u_X^{\text{ref}} & \text{if } \ t \in [0, T_X] \\ u_X^{\text{ref}} & \text{if } \ t > T_X, \end{cases}$$

  where $\alpha_X \in \{0.1, 0.5, 0.75\}$ and $T_X \in \{7, 14, 30\}$ for $X \in \{E, I\}$. If the effort to massive tests is not high, then $\alpha_E > \alpha_I$ and $T_I > T_E$.

4. Simulations

Given a vector of baseline parameters $P = (p, \mu_b, \mu_m, \gamma, \phi, u^{\text{ref}})$ (see Table 1), and an initial condition $(S_0, E_0, I_0, R_0, H_0, H_0^c, D_0)$ (see Table 2), we obtain the number of total deaths due to COVID-19 (D_{total}), the maximal demand of hospitalized in non complex services ($H_{\max}$), the maximal demand of critical facilities ($H^c_{\max}$) and the time where this demand is reached ($t_{\max}$). These values are reported in Table 3 and Figure 2.

D_{total}	$H_{\max}$	$H^c_{\max}$	$t_{\max}$ (days)
36922	45761	13569	97

Table 3. Results for baseline scenario.

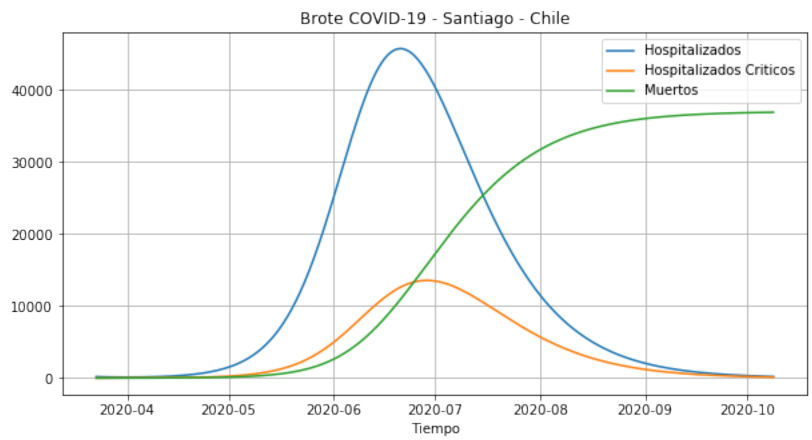

Figure 2. Baseline scenario April-October 2020. Without mitigation strategies, the demand of critical health service will be above 10 thousand units and the number of deaths above 5 thousands in mid June.

In this report we only simulate strategies of type *testing, isolation and contact tracing*. In future versions we will include simulations of other strategies described in Section 3.

For different values of $\alpha_X \in \{0.1, 0.5, 0.75\}$ (factor of reduction of contacts with exposed and infected people) we consider $u_X(t) \equiv \alpha_X u_X^{\text{ref}}$. The values obtained are presented in Table 4 where we also report the percentage reduction of maximal critical demand with respect the baseline scenario indicated in Table 3.

α_E	α_I	D_{total}	$H_{\max}$	$H^c_{\max}$	% gain	$t_{\max}$ (days)
1	0.5	14989	14261	4455	67%	169
0.75	0.75	26367	23961	7383	46%	145
0.75	0.5	3204	4701	1347	90%	200
0.5	0.5	463	309	93	99%	200

Table 4. Results for different interventions of reducing contacts with exposed and infected people. Reducing 50% the contact with exposed and 25% with infected, deaths are reduce drastically.

5. Final remarks

- The total number of deaths obtained (only for Santiago) from our model in the baseline scenario is very high, but is according to other studies in different cities as in [2].
- The type of model presented allows to estimate the magnitude order of maximal demands, but it is not appropriate for deducing an accurate estimation of daily cases.
- The strategies simulated are not easy to interpret. Reducing the rate of contact constantly we interpret as testing, isolation and contact tracing, because in this scenario the contact of general population with exposed and infected people decrease. Of course complete isolation of all population in their houses also reduce these rates of contact but these measures can not implemented all the time. In next reports we will simulate, based in the same model (3), other classes of strategies, as *distancing measures* (closing schools, universities, quarantines) as explained in 3 or feedback strategies as such tested in [2]. We think that some of the promising results showed in Table 4 can be replicated with temporally measures. On the other hand, the reduction of the rate of contacts in a factor α_X is not straightforward to interpret. The idea to use this factor is to show qualitatively where different efforts can imply a higher impact.
- Simulations for other cities can be easily implemented.

References

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